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ANALYSIS OF THE FLEXURAL VIBRATIONS OF A BEAM DUE TO THE MOTION OF A LINE LOAD†

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An analytic solution of a model problem of the flexural vibrations of a beam on an elastic Winkler foundation due to the front of a line load which moves along it is constructed. Quantitative results are presented for the special case when the velocity of the front is constant and the linear load is a step function. It is shown that a critical velocity of motion of the load exists and that, when this is exceeded, the elastic vibrations increase considerably. In this case, the dynamic range of deflection of the beam may be more than twice the magnitude of the displacement under the corresponding static load. The value of the critical velocity is determined by the mechanical properties of the beam and foundation and can be calculated using the ideal theory of an infinite beam. The amplitude of the deflection wave on approaching the critical velocity becomes larger as the length of the beam increases.

The analytic solution of a model problem of the flexure of an infinite beam lying on a continuous homogeneous elastic foundation due to a concentrated load moving along it at a constant velocity is well known [1, 2]. This solution is constructed in the form of a travelling wave and it has been shown that a critical velocity of the motion of the load exists and resonance amplification of the wave occurs when this velocity is reached.

1. FORMULATION OF THE PROBLEM

Consider a beam on an elastic base. A source of a line load with constant intensities Q_1 and Q_0 before and after the source, respectively, moves along this beam according to the law $x = x_*(t), x_*'(t) = V$. It is assumed that the cross-section and the modulus of elasticity E of the beam are constant and that its ends are free. According to Winkler's hypothesis [3], the reaction of the elastic foundation r is proportional to the deflection $u(x, t)$ of the axis of the beam: $r = -ku$, where k is the stiffness of the foundation or the bedding coefficient [1].

In order to generalize the problem, we shall assume that the decay of vibrations in an elastic system is due to an internal resistive force F_f which is proportional to the rate of displacement of the beam: $F_f = -\eta\partial u/\partial t$, where η is a coefficient of proportionality which characterizes the internal friction. It is supposed that, up to the engagement and start of the motion of the source at the instant of time $t = 0$, the beam is at rest and undeformed. The differential equation of the curved axis of the beam, together with the boundary and initial conditions, then takes the form

$$m\frac{\partial^2 u}{\partial t^2}+\frac{\partial^2}{\partial x^2}\left(\frac{EJ}{R}\right)+ku+\eta\frac{\partial u}{\partial t}=F(x,t)=[Q_0\Theta(x_*-x)+Q_1\Theta(x-x_*)]\Theta(t) \tag{1.1}$$

$$x=0,\quad x=l:\quad u_{xx}=u_{xxx}=0;\quad u(x,0)=0,\quad u_t(x,0)=0 \tag{1.2}$$

where x is the longitudinal coordinate, J and l are the moment of inertia of the planar cross-section of the beam and the length of the beam, respectively, m is the mass per unit length of the beam, R is the radius of curvature of its curved axis and Θ is the Heaviside function. If the treatment is confined to small deflections, we can put $1/R = u_{xx}$.

2. THE SELF-SIMILAR SOLUTION

If the velocity of motion of the source is constant and the length of the beam is infinite, the self-similar variable $\xi = x - Vt$ can be introduced and we can put $u(x, t) = f(\xi)$. From (1.1) and (1.2), we then obtain

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$$f'''' + 2bf'' - af' + \omega^2 f = q_0\Theta(-\xi) + q_1\Theta(\xi) \tag{2.1}$$
$$f''(\pm\infty) = f'''(\pm\infty) = 0$$
$$2b = \frac{mV^2}{EJ}, \quad \omega^2 = \frac{k}{EJ}, \quad q_0 = \frac{Q_0}{EJ}, \quad q_1 = \frac{Q_1}{EJ}, \quad a = \frac{\eta V}{EJ}$$

For simplicity, we shall neglect the losses of elastic energy, that is, we put $\eta = a = 0$. We obtain the solution of (2.1) by matching the deflections ahead of the source and behind it on the travelling wave front ($\xi = 0$). Conditions for the smooth matching of f up to the third derivative must be ensured [3].

We denote the self-similar solution, corresponding to $q_1 = 0$ by $\vartheta(\xi,)f_0$, where $f_0 = Q_0/k$. From (2.1), we then find

$$\vartheta(\xi) = \begin{cases} \theta^+(\xi), & \xi \geqslant 0 \\ 1 + \theta^-(\xi), & \xi \leqslant 0 \end{cases} \tag{2.2}$$
$$\theta^\pm(\xi) = \frac{1}{2}e^{\mp\alpha\xi}\left[\pm\cos\beta\xi + \frac{\alpha^2 - \beta^2}{2\alpha\beta}\sin\beta\xi\right]$$
$$\alpha^2 = \frac{m[V_c^2 - V^2]}{4EJ}, \quad \beta^2 = \frac{m[V_c^2 + V^2]}{4EJ}, \quad V_c^4 = \frac{4kEJ}{m^2}$$

This is valid for $V < V_c$. When $V \to V_c$, we have $\alpha \to 0$, $\vartheta(\xi,) \to \infty$, if $\xi \neq 0$. If, however, $V > V_c$, a self-similar solution does not exist.

We next analyse the deflection of the beam when $q_1 \neq 0$.

Theorem 1. The self-similar solution f when $q_1 \neq 0$ is expressed in terms of the normalized deflection of the beam when ϑ when $q_1 = 0$ according to the formula: $(f - f_1) = (f_0 - f_1)\vartheta(\xi), f_1 = Q_1/k$.

For the proof, consider the function $\varphi = f - f_1$. From (2.1), we obtain $2b\varphi'' + \omega^2\varphi + \varphi'''' = (q_0 - q_1)\Theta(-\xi)$, that is, the problem with $Q_1 \neq 0$ is reduced to the previously investigated problem when $Q_1 = 0$ but with Q_0 replaced by $Q_0 - Q_1$.

It follows from the theorem that $f(\xi) + f(-\xi) = f_0 + f_1, f(0) + (f_0 + f_1)/2$.

A plot of the analytic solution is shown in Fig. 1, where $f = (f - f_1)/(f_0 - f_1)$, $\xi^- = \sqrt{(m/(4EJ))}V_c\xi$. Curves 1–3 correspond to values of V/V_c equal to 0, 0.9 and 0.99, respectively. It is clear that the normalized deflection $\vartheta(\xi) - 1/2$ is an odd function of ξ. The external values ϑ_*, which are attained at the points ξ_*, are distributed according to the law

$$\beta\xi_{*-}^{(j)} = \mathrm{arctg}(\beta/\alpha) - \pi j < 0, \quad j = 1, 2, 3, \ldots$$
$$\vartheta(\xi_{*-}^{(j)}) = 1 + \tfrac{1}{4}(-1)^{j+1}\exp(\alpha\xi_{*-}^{(j)})[1 + (\beta/\alpha)^2]^{\frac{1}{2}} \tag{2.3}$$

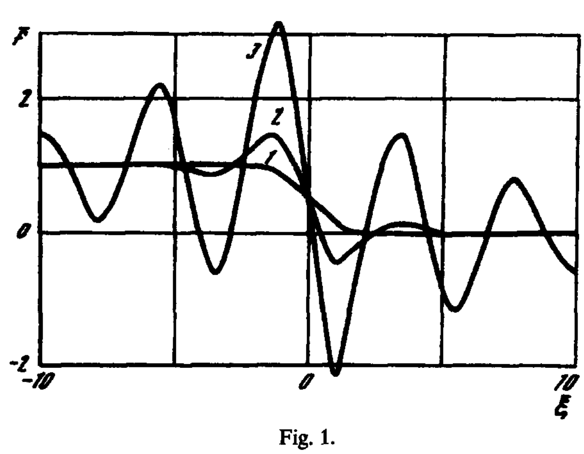

Fig. 1.

When $\xi_* > 0$, we have

$$\xi_* = \xi_{*+}^{(j)} = -\xi_{*-}^{(j)}, \quad \vartheta(\xi_{*-}^{(j)}) - \vartheta(0) = \vartheta(0) - \vartheta(\xi_{*+}^{(j)})$$

The equality

$$\bar{\xi}_{*-}^{(j)} = \pi(1/4 - j) \tag{2.4}$$

holds in the case of a static load ($V = 0$).

As the critical velocity ($V = V_c$) is approached, the expression on the left-hand side of equality (2.4) tends to the limit $\pi(1/2 - j)/\sqrt{2}$. As the velocity V increases, the extremal points approach the source ($\xi = 0$). In view of (2.3), the distance between all the neighbouring extremal points, with the exception of those closest to the source, is the constant quantity: $|\xi_*^{(j+1)} - \xi_*^{(j)}| = \pi/\beta$.

For $j = 1$, we have $|\xi_{*+}^{(1)} - \xi_{*-}^{(1)}| = 2[\pi - \text{arctg}(\beta/\alpha)]/\beta$. As the velocity V increases, the distance between all the neighbouring antinodes contracts to a value of $\pi/[\sqrt{(m/(2EJ))}V_c]$. At low velocities $\upsilon \ll \upsilon_c$, the amplitude normalized values of the wave of the deflections ϑ_* are close to unity, that is, the static load approximation is acceptable. The effect of a dynamic load shows up most clearly when $V \to V_c$ and, then, $|\vartheta_*| \to \infty$.

The distribution of the nodes of the flexural wave, that is, the zeros of the function $\vartheta(\xi)$ when $\xi > 0$: $\vartheta(\xi_0^{(j)}) = 0$, is traced in a similar manner to its antinodes. From (2.2), we obtain

$$\left[1 + \left(\frac{V}{V_c}\right)^2\right]^{1/2} \bar{\xi}_0^{(j)} = \pi j - \text{arctg}\left[\left(\frac{V_c}{V}\right)^4 - 1\right]^{1/2}$$

3. A BEAM OF FINITE LENGTH

A self-similar elastic wave exists for a source velocity $V < V_c$ in the case of an infinite beam. When the beam length is finite, this solution will be acceptable far from its ends, if $\alpha l \gg 1$, $\beta l \gg 2\pi$. When these conditions are violated, the solutions must be investigated taking account of the reflection of elastic waves from the boundaries $x = 0, x = l$.

Theorem 2. The boundary-value problem (1.1), (1.2), which describes the vibrations of a beam with an arbitrary moving line load F, can only have a unique solution for small deflections.

Proof. Let us assume that two different solutions exist. Then, for their difference, we have

$$m\delta u_{tt} + \eta\delta u_t + k\delta u + EJ\delta u_{xxxx} = 0 \tag{3.1}$$

$$\delta u(x, 0), \quad \delta u_t(x, 0) = 0$$

$$\delta u'(0, t) = \delta u'''(0, t) = 0, \quad \delta u''(l, t) = \delta u'''(l, t) = 0$$

Let us multiply Eq. (3.1) by δu_t and integrate with respect to x from 0 to l and with respect to t, from 0 to t, using the boundary and initial conditions. As a result, we obtain a relation from which it follows that $\delta u = 0$.

Since (1.1) is a linear partial differential equation with constant coefficients, its solution is conveniently sought in the form of Fourier series in the eigenfunctions of the homogeneous ($F = 0$) stationary problem. Formally, the solution can be found by the method of separation of variables

$$u(x,t) = \sum_{k=0}^{\infty} \chi_k(x)\nu_k(t), \quad F(x,t) = \sum_{k=0}^{\infty} \Phi_k(t)\chi_k(x) \tag{3.2}$$

Substituting (3.2) into (1.1) and (1.2), we obtain an eigenvalue boundary-value problem for the eigenvalues $\gamma = \gamma_n$ ($n = 0, 1, 2, \ldots$).

$$\chi_n'''' = \gamma_n^4 \chi_n, \quad m\ddot{\nu}_n + \eta\dot{\nu}_n + (k + EJ\gamma_n^4)\nu_n = \Phi_n \tag{3.3}$$

$$x = 0, \quad x = l: \ \chi_n'' = \chi_n''' = 0; \quad t = 0: \ \nu_n = \nu_n' = 0$$

The general solution has the form

$$\chi(x) = A\text{ch}\gamma x + B\text{sh}\gamma x + C\cos\gamma x + D\sin\gamma x; \quad A, B, C, D = \text{const}$$

Using the boundary conditions, we obtain the characteristic equation $\text{ch}\,\mu\cos\mu = 1$, where $\mu = \gamma l$. The discrete spectrum $(\mu = \mu_n)$: $\mu 0 = 0$, $\mu_{2k-1} \in [\pi(4k-1)/2, 2\pi k]$, $\mu_{2k} \in [2\pi k, \pi(4k+1)/2]$, $k \gg 1$. If $n \gg 1$, then $\mu_n \sim u_n^a \equiv \pi(n + 1/2)$.

When $n \geqslant 1$, all the eigenvalues are single.

If, however, $n = 0$, the root μ is double and there are two independent eigenfunctions which correspond to it. It follows from (3.3) that $\chi_0 = Gx + H$, where $G, H = \text{const}$. One of the two eigenfunctions can be set equal to a constant: $\chi_{01} = A_{01} = \text{const}$. The other (χ_{02}) is found from the condition that it is orthogonal to χ_{01}.

It can be shown that all of the eigenfunctions χ_k, which correspond to different eigenvalues μ_k, are orthogonal. On additionally imposing the normalization condition $\int_0^l \chi_k^2(x)dx = 1$ on them, we obtain

$$\chi_n(x) = A_n[\text{ch}\gamma_n x + \cos\gamma_n x - C_n(\text{sh}\gamma_n x + \sin\gamma_n x)] \tag{3.4}$$

$$C_n = (\text{ch}\mu_n - \cos\mu_n)/(\text{sh}\mu_n - \sin\mu_n)$$

$$A_n = \{(1 + C_n^2)N_1 + (1 - C_n^2)N_2 - 2C_n(N_3 + N_4)\}^{-1/2}$$

$$N_1 = l\left[\frac{\text{sh}\,2\mu_n}{4\mu_n} + \frac{1}{2}\right], \quad N_2 = l\left[\frac{\sin 2\mu_n}{4\mu_n} + \frac{1}{2}\right]$$

$$N_3 = l\frac{1 - \cos 2\mu_n}{4\mu_n}, \quad N_4 = \frac{\text{ch}\,2\mu_n - 1}{4\mu_n}, \quad n \geqslant 1$$

$$\chi_{01}(x) = A_{01} = 1/\sqrt{l}; \quad \chi_{02}(x) = A_{02}(1 - 2x/l), \quad A_{02} = \sqrt{3/l} \tag{3.5}$$

The eigenvalues μ_n, obtained numerically $(n \geqslant 1)$ are: $\mu_1 = 3\pi/2 + 1.76551 \times 10^{-2}$, $\mu_2 = 5\pi/2 - 7.7763 \times 10^{-4}$, $\mu_3 = 7\pi/2 + 3.2712 \times 10^{-5}$, $\mu_4 = 9\pi/2 - 1.9412 \times 10^{-6}$, $|\mu_n - \pi(n + 1/2)| < 6 \times 10^{-7}$ when $n \geqslant 5$. The coefficients A_n and C_n are presented below

n	1	2	3	4	5	6	7	8	9
$(A_n\sqrt{l} - 1) \times 10^6$	0	−1	−1	−1	−1	−209	10782	9495	8493
$(C_n - 1) \times 10^6$	−17498	777	−34	1	−1	0	−1	0	−1

n	10	11	12	13	14	15	16	17	18
$(A_n\sqrt{l} - 1) \times 10^6$	7666	6992	6427	5947	5533	5173	4858	4578	4329

$|C_n - 1| < 5 \cdot 10^{-7}$ when $n \geqslant 10$.

Plots of the eigenfunctions are shown in Fig. 2. In view of the double degeneracy of the least eigenvalue $\mu_0 = 0$, the number $n = 0$ which corresponds to it is labelled as 0_* and 0_{**} for the first (χ_{01}) and the second (χ_{02}) eigenfunctions, respectively.

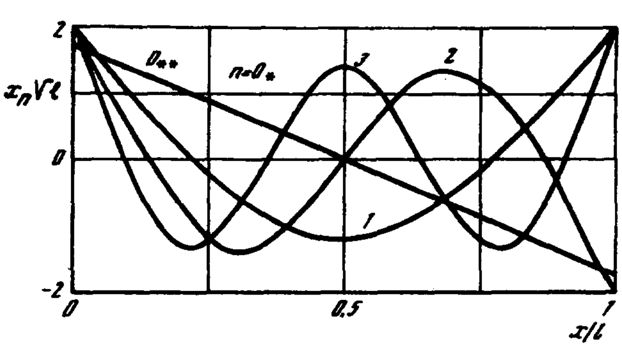

Fig. 2.

It is clear that, when $n = 0_*$, the eigenfunction is even with respect to the centre of the beam ($x = l/2$) and does not have zeros. Subsequently, each eigenfunction following in number (0_{**}, 1, 2, and so on) changes its parity into the opposite parity and additionally acquires some zeros.

Having the orthonormal basis $\chi_n(x)$, to construct the solution of the general problem on the displacements of beams, it is still necessary to find the coefficients of the expansion with respect to the basis $v_n(t)$. It can be shown that, in the general case for an arbitrary line load $F(x, t)$

$$v_n(t)=\frac{1}{ml\psi_0 l}\int_0^t K(t-\tau)\Phi_n(\tau)d\tau \tag{3.6}$$

$$K(\zeta)=\exp\left(-\frac{\eta}{2m}\zeta\right)\times\begin{cases}\sin\psi_n\zeta, & \eta\leq\eta_c\\ \operatorname{sh}|\psi_n|\zeta, & \eta\geq\eta_c\end{cases}$$

$$\psi_n=\left[\frac{k+EJ(\mu_n/l)^4}{m}-\left(\frac{\eta}{2m}\right)^2\right]^{1/2}, \quad \psi_n(\eta_c)=0$$

$$\Phi_n(t)=\int_0^l F(x,t)\chi_n(x)dx$$

Next, in order to simplify the analysis, we consider the special case of the equilibrium motion of a source $x_* = Vt$, $V = \text{const}$ with a line load $F(x, t)$ which differs from the corresponding quantity defined in (1.2) by the factor $\beta(t)$ which takes account of the possibility of the disengagement of the load when the source leaves the beam at the right-hand end: $\beta(t) = 1$ if the load does not disappear when $x_* > l$ and $\beta(t) = \Theta(l-x_*)$ if the load is fully disconnected when the source leaves the beam.

After calculating the integrals, we obtain

$$\Phi_{01}(t)=\begin{cases}A_{01}[Vt(Q_0-Q_1)+Q_1 l], & t\leq t_*=l/V\\ A_{01}\beta_0 Q_0 l, & t\geq t_*,\ n=0_*\end{cases}$$

$$\Phi_{02}(t)=\begin{cases}A_{02}l(Q_0-Q_1)(t-t_*)^2, & t\leq t_*\\ 0 & t\geq t_*,\ n=0_{**}\end{cases}$$

$$\Phi_n(t)=\begin{cases}Q_0[\alpha(Vt)-\alpha(0)]+Q_1[\alpha(l)-\alpha(Vt)], & x_*\leq l\\ \beta_0 Q_0[\alpha(l)-\alpha(0)], & x_*\geq l,\ n\geq 1\end{cases}$$

$$\alpha(x)\equiv\int_0^x \chi_n(x)dx+\alpha(0)=A_n[\operatorname{sh}\gamma_n x+\sin\gamma_n x-C_n(\operatorname{ch}\gamma_n x-\cos\gamma_n x)]/\gamma_n$$

where $\beta_0 = 1$, if the load remains when the source leaves the beam and $\beta_0 = 0$ otherwise.

For the coefficient $v_{01}(t)$ of the expansion with respect to the first eigenfunction, we obtain

$$m\psi_0 v_{01}(t)=A_{01}\{(Q_0-Q_1)I_{v1}(s,t)+Q_1 lI_1(s,t)+\beta_0 Q_0 l[I_1(t,t)-I_1(s,t)]\};$$

$$I_1(s,t)=\Omega[I_{11}(t-s)-I_{11}(t)]$$

$$I_{v1}(s,t)=\int_0^s K(t-\tau)V\tau d\tau=V\left\{\left(s-\frac{\eta\Omega}{m\psi_n}\right)I_1(s,t)+\Omega sI_{11}(t)+\Omega\frac{[K(t-s)-K(t)]}{\psi_n}\right\},$$

$$n=0;\quad \Theta(0)=\frac{1}{2}$$

$$s=t\Theta(t_*-t)+t_*\Theta(t-t_*),\quad \Omega=\left\{\psi_n\left[1+\left(\frac{\eta}{2m\psi_n}\right)^2\right]\right\}^{-1}$$

$$I_{11}(t)=\exp\left(-\frac{\eta t}{2m}\right)\left[\cos\psi_n t+\frac{\eta\sin\psi_n t}{2m\psi_n}\right]$$

It is obvious that $\alpha(0) = 0$. Next, when $n \geq 1$, we have $\alpha(l) = 0$ since the constant is an eigenfunction and they are orthogonal in the space $L_2[0, l]$.

The second coefficient of the expansion of the solution $v_{02}(t)$ in the basis of orthonormalized eigenfunctions $\chi_n(h)$ has the form

$$m\psi_0 v_{02}=A_{02}(Q_0-Q_1)\{I_{v1}(s,t)-I_{v2}(s,t)/l\}$$

$$I_{v2}(s,t)=\int_0^s K(t-\tau)(V\tau)^2 d\tau = V\{sI_{v1}(s,t)-\kappa(s,t)\}$$

$$\kappa(s,t)=\int_0^s I_{v1}(\tau,t)d\tau = V\{s\Phi_n(s,t)-H_n(s,t)-H_0(s,t)\}$$

$$\Phi_n(s,t)=\int_0^s I_1(\tau,t)d\tau=\Omega\cdot\left\{-sI_{11}(t)+\frac{\eta}{m\psi_n}I_1(s,t)+\frac{K(t)-K(t-s)}{\psi_n}\right\};$$

$$H_n(s,t)=\int_0^s \Phi_n(\tau,t)d\tau=\Omega\left\{-\frac{s^2}{2}I_{11}(t)+\frac{\eta}{m\psi_n}\Phi_n(s,t)+s\frac{K(t)}{\psi_n}-\frac{I_1(s,t)}{\psi_n}\right\}$$

The coefficients of the expansion $v_n(t)$ when $n \geqslant 1$ are determined as

$$m\psi_n v_n(t)=(Q_0-Q_1)\frac{A_n}{\gamma_n}\{I_2(s,t)+I_4^+(s,t)-C_n[I_3(s,t)-I_4^-(s,t)]\}$$

$$I_2(s,t)=\frac{1}{2\psi_n}\{-I_{22}(-V,t-s)e^{-\gamma_n Vs}+I_{22}(-V,t)-I_{22}(+V,t)+I_{22}(+V,t-s)e^{\gamma_n Vs}\},$$

$$a(V)=\frac{\eta}{2m\psi_n}+\frac{\gamma_n V}{\psi_n}$$

$$I_{22}(V,t)=\frac{\exp[-\eta t/(2m)]}{a^2(V)+1}[\cos\psi_n t+a(V)\sin\psi_n t]$$

$$I_3(s,t)=\frac{1}{2\psi_n}\left\{I_{22}(-V,t-s)e^{-\gamma_n Vs}-I_{22}(-V,t)+I_{22}(+V,t-s)e^{\gamma_n Vs}-I_{22}(+V,t)\right\}$$

$$I_4^{\pm}(s,t)=[I^{\pm}(s,t)\cos\gamma_n Vt\pm I^{\mp}(s,t)\sin\gamma_n Vt]/\psi_n$$

$$I^{\pm}(s,t)=[c\Phi^{\mp}(s,t)\mp d\Phi^{\pm}(s,t)]/(c^2+d^2)$$

$$c=1+\left(\frac{\eta}{2m\psi_n}\right)^2-\left(\frac{\gamma_n V}{\psi_n}\right)^2,\quad d=2\left(\frac{\eta}{2m\psi_n}\right)\left(\frac{\gamma_n V}{\psi_n}\right)$$

$$\Phi^{\pm}(s,t)=\Psi^{\pm}(t-s)-\Psi^{\pm}(t)$$

$$\Psi^{\pm}(t)=\exp\left(-\frac{\eta t}{2m}\right)+\left[\pm a^{\pm}(t)\cos\psi_n t+\frac{\gamma_n V a^{\mp}(t)\pm(\eta/(2m))a^{\pm}(t)}{\psi_n}\sin\psi_n t\right]$$

$$a^+(t)=\cos\gamma_n Vt,\quad a^-(t)=\sin\gamma_n Vt$$

The analytic solution obtained above is quite complex, and certain special cases can be considered. They are useful for testing the analytic and numerical calculations of more general problems. Let the load be homogeneous: $Q_0 = Q_1$. If we denote the corresponding solution by $u_0(x, t)$, then $v_n(t) \equiv 0$ when $n \geqslant 1$, $v_{02} \equiv 0$ and

$$m\psi_0 v_{01}=A_{01}\cdot l\cdot Q_0\{\beta_0 I_1(t,t)+(1-\beta_0)I_1(s,t)\}$$

$$u_0(x,t)=f_0[U(p)-U(t)]$$

$$U(t)=\left[\cos(\psi_0 t)+\frac{\eta}{2m\psi_0}\sin(\psi_0 t)\right]\exp\left(-\frac{\eta t}{2m}\right)$$

where $p = (1-\beta_0)(t-s)$; $s = t$ when $t \leqslant t_*$, $s = t_*$ when $t \geqslant t_*$ and $f_0 = Q_0/k$ is the amplitude of the elastic wave.

It can be shown that the solution of the general problem in the case of an inhomogeneous line load ($Q_0 \neq Q_1$) will tend to it with time.

In the calculations, unless otherwise stated, we have taken $V/V_c = 0.73$; $Q_1 = 0$, $\eta = 0$; $EJ/(kl^4) = 1.16\times10^{-6}$; $mV_c^2(kl^2) = 2.2\times10^{-3}$. The calculations showed that it is sufficient to use 15–20 terms of the series (3.2).

The effect of the parameter η, which characterizes the internal friction of the elastic medium, on the displacement of the beam is shown in Fig. 3. Here, $V/V_c = 0.5$ and $\beta_0 = 0$. The dependence of the deflection on the longitudinal coordinate x and on time t is shown respectively, for $Vt/l = (1 + 2\times10^{-4}V/l)/3$ and for the middle part of the beam ($x = l/2$). Curves 1 and 2 correspond to values of the parameter $\bar{\eta} = \eta l/mV$ of 0 and 10, respectively. When

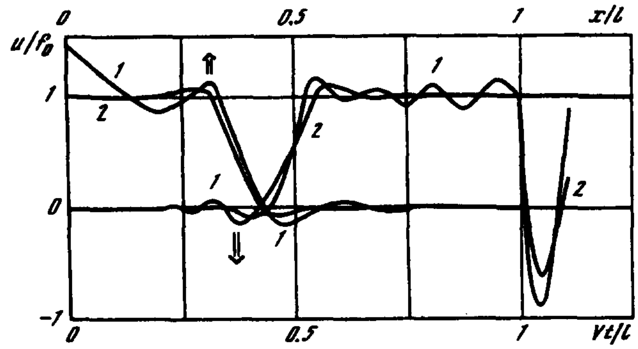

Fig. 3.

$\bar{\eta} \sim 10$, the deflection u is significantly smoothed out, approaching the static deflection when $u = f_1$ at a sufficient distance in front of the source and $u = f_0$ behind it. The amplification of the vibrations when $Vt/l \gg 1$ is due to the removal of the load when the source flies off with it. If $\bar{\eta}$ is of the order of unity, the calculated curves are close to those for deflections of a beam when there is no friction ($\eta = 0$).

Figures 4 and 5 are representations of the deflection as a function of the different modes of the front of the load, "subsonic" ($V < V_c$), "transonic" ($V \approx V_c$) and "supersonic" ($V > V_c$). Curves 1–3 correspond to values of V/V_c of 0.5, 1.0 and 1.5.

Figure 4 shows the dependence of the deflection on time. The solid curves correspond to $\beta_0 = 0$ and the dashed curves to $\beta_0 = 1$.

The distribution of the deflection along the beam is shown in Fig. 5. It is seen that, regardless of the acceleration mode, the maximum deflection of the beam is found behind the load front. On changing to "supersonic" conditions, there is an increase in the amplitude of the elastic wave. It may be more than twice the magnitude of the displacement of the beam under static loading. The displacements ahead of the front are an order of magnitude or more smaller than the deflections behind it. Under supersonic conditions, the deformations in front of the source are reduced considerably. However, intense vibrations now occur behind it. An unlimited increase in the amplitude of the elastic wave on attaining the critical velocity, which is predicted by the self-similar solution, is not observed.

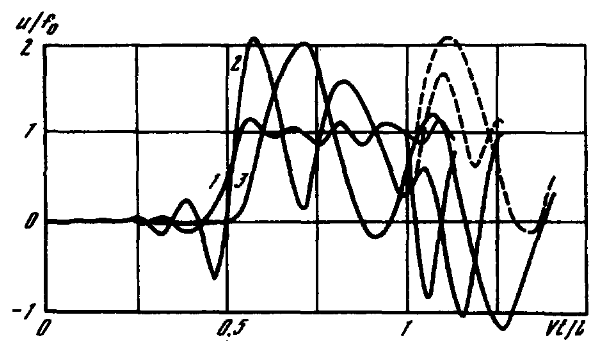

Fig. 4.

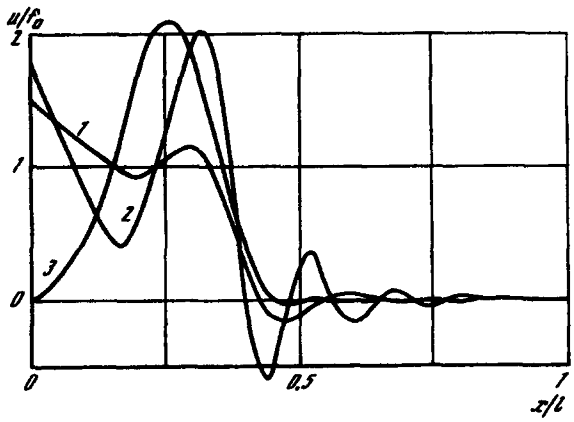

Fig. 5.

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